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CLASS: XII

MID TERM EXAM (SESSION: 2023– 24)

SUBJECT: APPLIED MATHEMATICS (CODE 241)

SET B

Time Allowed: 3 Hrs. **Max. Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there is some internal choice in some questions.
2. Section A has 18 MCQ's and 02 Assertion Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA) questions of 2 marks each.
4. Section C has 6 Short Answer (SA) questions of 3 marks each.
5. Section D has 4 Long Answer (LA) questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (04 marks each) with sub parts.
7. Internal Choice is provided in 2 questions in Section-B, 2 questions in Section-C, 2 Questions in Section-D. You have to attempt only one alternative in all such questions.

SECTION A

(All Questions are compulsory. No internal choice is provided in this section)

Q1. The solution of the differential equation $\frac{dx}{x} + \frac{dy}{y} = 0$ is

(a) $\frac{1}{x} + \frac{1}{y} = C$ (b) $x + y = C$ (c) $logx\ logy = C$ (d) $xy = C$

Q2. If $\int_0^{40} \frac{dx}{2x+1} = \log k$, then the value of k is

(a) 4 (b) 3 (c) 9 (d) 1

Q3. If the radius of a circle is increasing at the rate of 2 cm/sec, then the area of the circle when its radius is 20 cm is increasing at the rate of

(a) $80\pi\ m^2/sec$ (b) $80\ m^2/sec$ (c) $80\pi\ cm^2/sec$ (d) $80\ cm^2/sec$

Q4. $\int \frac{(\log x)^5}{x} dx$ is equal to

(a) $\frac{\log x^6}{6} + C$ (b) $\frac{(\log x)^6}{3x^2} + C$ (c) $\frac{\log x^6}{3x^2} + C$ (d) $\frac{(\log x)^6}{6} + C$

Q5. If $\frac{x+1}{x+2} \geq 1, then$

(a) $x \in [-\infty, 2]$ (b) $x \in (-\infty, -2)$

(c) $x \in (-\infty, 2]$ (d) $x \in (-\infty, 2)$

Q6. A vehicle costing Rs. 125000 has scrap value of Rs. 25000. If annual depreciation charge is Rs. 12500, then useful life of the vehicle is

(a) 4 years (b) 6 years

(c) 8 years (d) 10 years

Q7. $\int e^x \left(\frac{1}{x^2} - \frac{2}{x^3}\right) dx$ is equal to

(a) $\frac{e^x}{x^2} + C$ (b) $\frac{e^x}{x^3} + C$ (c) $\frac{-e^x}{x^2} + C$ (d) $\frac{e^x}{2x^3} + C$

Q8. If $C(x)$ and $R(x)$ are respectively Cost function and Revenue function, then profit function $P(x)$ is given by

(a) $P(x) = R(x)$ (b) $P(x) = C(x) + R(x)$

(c) $P(x) = R(x) - C(x)$ (d) $P(x) = R(x).C(x)$

Q9. The least non-negative remainder, when 3^{15} is divided by 7 is

(a) 5 (b) 1 (c) 6 (d) 7

Q10. The equation of normal at the point (1,1) to the curve $2y + x^2 = 3$ is

(a) $x + y = 0$ (b) $x - y = 0$

(c) $x + y = 1$ (d) $x - y = 1$

Q11. The ratio in which a grocer mixes two varieties of pulses costing Rs. 85 per kg and Rs. 100 per kg respectively so as to get a mixture worth Rs. 92 per kg is

(a) 8: 7 (b) 7: 8 (c) 5: 7 (d) 7: 5

Q12. For what value of x, is the following matrix singular? $\begin{bmatrix} 3-2x & x+1 \\ 2 & 4 \end{bmatrix}$

(a) $x = -1$ (b) $x = 2$ (c) $x = 1$ (d) $x = -2$

Q13. Region represented by $x \geq 0, y \geq 0$ lies in

(a) I quadrant (b) II quadrant (c) III quadrant (d) IV quadrant

Q14. Let $p > 0$ and $q < 0$ and $p, q \in Z$, then choose the correct inequality from the given below options to complete the statement: $p + q$ ____ $p - q$

(a) > (b) ≤ (c) ≥ (d) <

Q15. If $\begin{bmatrix} x+3y & y \\ 7-x & 4 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ 0 & 4 \end{bmatrix}$, then the values of x and y are:

(a) $x = 7, y = -1$ (b) $x = -7, y = -1$

(c) $x = 7, y = 1$ (d) $x = -7, y = 1$

Q16. If the value of the objective function Z can be increased or decreased indefinitely, such solution is called…….

(a) Bounded solution (b) Optimum solution

(c) Unbounded solution (d) Feasible solution

Q17. An investment of ₹ 5,000 becomes ₹ 25,000 in 4 years, then the CAGR (compound annual growth rate) is given by –

(a) $\left[\sqrt[4]{5}-1\right]\times 100$ (b) $\left[\sqrt[4]{5}+1\right]\times 100$

(c) $\frac{\sqrt[4]{5}-1}{100}$ (d) $\frac{\sqrt[4]{5}+1}{100}$

Q18. A and B are square matrices each of order 3 such that $|A|=-1$ and $|B|=3$. What is the value of $|3AB|$?

(a) -27 (b) -9 (c) -18 (d) -81

For questions 19 and 20 two statements are given – one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to the question from the codes (i), (ii), (iii) and (iv) as given below:

(i) Both A and R are true and R is the correct explanation of the assertion

(ii) Both A and R are true but R is not the correct explanation of the assertion

(iii) A is true, but R is false

(iv) A is false, but R is true

Q19. Assertion (A): $\int_2^3 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{5-x}}dx=\frac{1}{2}$

Reason (R): $\int_a^b f(x)dx=\int_a^b f(a+b-x)dx$

Q20. Assertion (A): The present value of a sequence of payments of Rs. 600 made at the end of each quarter and continuing forever, if money is worth 6% compounded quarterly is Rs. 10,000.

Reason (R): The present value of a perpetuity of Rs. R payable at the end of each period, the first being due one period hence is $P=\frac{R}{i}$ where R denotes size of each payment and i denotes rate per period.

SECTION B

(All Questions are compulsory. In case of internal Choice, attempt any one question only)

Q21. At what rate of interest will the present value of a perpetuity of Rs 500 payable at the end of every 6 months be Rs. 10,000?

OR

Find the effective rate of interest equivalent to a nominal rate of 6% compounded semi-annually.

Q22. In a 200-metre race, Anuj can beat Param by 5 meter or 3 seconds. How much time did Anuj take to complete the race?

OR

Find the remainder when (226 × 369 × 122 × 461 × 1025) is divided by 8.

Q23. Find the least value of a so that the function $f(x) = x^2 + ax + 1$ is strictly increasing on [1,2].

Q24. If $X = \begin{bmatrix} -1 & 3 \\ 8 & 4 \end{bmatrix}$ and $Y = \begin{bmatrix} -5 & 1 \\ -1 & -2 \end{bmatrix}$ then find the matrix Z such that $2X + Y = 5Z$.

Q25. A company has two groups of inspectors namely, group A and B, who are assigned to do a quality inspection work. It is required that at least 1800 pieces are inspected for 8-hour day. It is known that inspectors of group A can check pieces at the rate of 25 per hour with an accuracy of 98%, while inspectors of group B can check at the rate of 15 pieces per hour with an accuracy of 95%. The inspectors of group A and B are paid Rs 40 and Rs 30 per hour respectively to do the work. Each time an error is caused by the any inspector, it costs a loss of Rs 20 to the company. The company has 8 inspectors in group A and 10 in group B. The company wants to determine the optimal assignment of Inspectors to minimize total inspection cost. Formulate an LPP.

SECTION C

(All Questions are compulsory. In case of internal Choice, attempt any one question only)

Q26. A cistern can be filled by two pipes A and B in 12 minutes and 15 minutes respectively. Another tap C can empty the full tank in 20 minutes. If the tap C is opened 5 minutes after the pipes A and B are opened, find when the cistern becomes full?

OR

A boat covers 32 km upstream and 36 km downstream in 7 hours. Also it covers 40 km upstream and 48 km downstream in 9 hours. Find the speed of the boat in still water and that of the stream.

Q27. Verify that y= $\frac{1}{x} - logx$ is a solution of the differential equation,

$$x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} - y = \log x$$

Q28. A machine costing Rs.2,00,000 has effective life of 7 years and its scrap value is Rs.30,000. What amount should the company put into a sinking fund earning 5% per annum, so that it can replace the machine after its useful life? Assume that a new machine will cost Rs. 3,00,000 after 7 years. (Given $(1.05)^7 = 1.407$)

Q29. If $x = \frac{1}{2}(e^t + e^{-t})$ and $y = \frac{1}{2}(e^t - e^{-t})$, show that $y^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$.

Q30. Express $\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix.

Q31. The demand and supply functions are $p_d = 25 - x^2$ *and* $p_s = 2x + 1$ respectively. Find the producer's surplus at equilibrium price.

OR

Evaluate $\int_0^2 x^2\sqrt{2-x}\,dx$

SECTION D

(This section comprises of long answer type questions (LA) of 5 mark each)

Q32. A metal box with a square base and vertical sides is to contain $1024\ cm^3$ of water. The material for the top and bottom costs Rs. 5 per cm^2 and the material for the sides costs Rs. 2.50 per cm^2. Find the least cost of the box.

OR

For a monopolist's product, the demand function is $p = \frac{50}{\sqrt{x}}$ and average cost function $AC = 0.5 + \frac{2000}{x}$. Find the profit maximizing level of output. At this level, show that the marginal revenue and marginal cost are equal.

Q33. The amount of radiocarbon present after t years is given by
$A = A_0 e^{-(\log 2)\left(\frac{1}{5700}\right)t}$, *where* A_0 is the amount present in the living plants and animals.
(i) Find the half-life of radiocarbon.
(ii) Charcoal from an ancient pit contained $\frac{1}{4}$ of the carbon-14 found in living sample of same size. Estimate the age of the charcoal.

Q34. A machine costing Rs. 50,000 depreciates at a constant rate of 8%. What is the depreciation charge for the 8th year. If the estimated useful life of the machine is 10 years, determine its scrap value.

Q35. A manufacturer has three machines I, II and III installed in his factory. Machines I and II are capable of being operated for atmost 12 hours whereas machine III must be operated atleast 5 hours a day. He produces only two item M and N each requiring the use of all the three machines. The number of hours required for producing 1 unit of each of M and N on the three machines are given in the following table:

Items	Number of hours required on machines		
	I	II	III
M	1	2	1
N	2	1	1.25

He makes a profit of ₹600 and ₹ 400 on each item of M and N respectively. How many units of each item should he produce so as to maximize his profit assuming that he can sell all the items that he produces? What will be the maximum profit?

OR

There are two types of fertilizers "A and B". 'A' consists of 12% nitrogen and 5% phosphoric acid where as 'B' consists of 4% nitrogen and 5% phosphoric acid. After testing the soil conditions, farmer finds that he needs at least 12 kg of nitrogen and 12 kg of phosphoric acid for his crops. If 'A' costs Rs. 10 per kg and 'B' costs Rs. 8 per kg, then using linear programming determine how much of each type of fertilizer should be used so that the nutrient requirements are met at a minimum cost?

SECTION – E

(This section comprises of 3 source-based questions (Case Studies) of 4 mark each)

Q36. **CASE STUDY – I -** (Internal Choice is in option iii.) (Mark 1 + 1 + 2)

A loan of Rs. 2,50,000 at the interest rate of 6% p.a. compounded monthly is to be amortized by equal payments at the end of each month for 5 years.

Based on the above information, answer the following questions. Show steps to support your answers.

(i) Find the size of each monthly payment

(ii) Find total interest paid

(iii) Find the principal outstanding at beginning of 40^{th} month.

OR

Find the interest paid in 40^{th} month.

(Given $(1.005)^{60} = 1.3489, (1.005)^{21} = 1.1104$)

Q37. **CASE STUDY – II -** (Mark 2+1+1) (Internal choice is in the iii part)

The marginal cost $(in\ Rs.)$ of a product is given by $MC = \frac{300}{\sqrt{3x+25}}$ and the fixed cost is $Rs.5000$.

Based on the above information, answer the following questions. Show steps to support your answers.

(i) Find the cost function.

(ii) Find the average cost function.

(iii) Find the cost of producing 25 units of the product.

OR

(iv) Find the average cost of producing 200 units of the product

Q38. **CASE STUDY III**

A school plans to award Rs. 6000 in total to its students to reward for certain values – honest, regularity and hard work. When three time the award money for hard work is added to the award money given for honesty amounts to Rs. 11,000. The award money for honesty and hard work together is double the award money for regularity. Use matrix method to find the prize money for each category of award.

OR

The sum of three numbers is 20. If we multiply the first number by 2 and add the second number to the result and subtract the third number, we get 23. By adding second and third numbers to three times the first number, we get 46. Represent the above problem algebraically and use Cramer's rule to find the numbers from these equations.